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THE REORGANIZATION OF THE MATHEMATICAL CURRICULUM IN SECONDARY SCHOOLS.*

BY PROF. ARTHUR D. PITCHER.

It is with considerable diffidence that I speak to you on the subject of the mathematical curriculum in the secondary schools. I am neither a teacher of secondary mathematics, nor an expert in the pedagogy of mathematics. However I have a permanent interest in the subject and its progress, and in the whole people to whom I believe the advancement of mathematics and mathematical instruction is fundamentally important. It is from this point of view that I speak. I cannot hope to give you an expert opinion on the curriculum, for I am not that kind of an expert. I can only register here my reaction on the subject in hand. It may well be doubted if my opinions are worth while. In fact I would not give them if I thought they would be accepted as given. I shall be thoroughly satisfied if I can emphasize a few well-known points of view, and perhaps stimulate a reasonable amount of criticism and thought.

The American secondary school has come to be a powerful factor in public education. It is indeed the people's college and the question whether it is doing what it should for them is a pertinent one. What then is the true end of the secondary school of to-day? I speak primarily of the public high school, and of those private schools who think of themselves as filling the same place in the educational system of the country as the public high school. No one can deny the right of a given private school to serve a special class of students. Such schools can hardly come within the scope of a general discussion of this kind. In particular I think we must agree with the oft-reiterated statement, that the purpose of the school of which we are speaking is not to prepare the student for college. I, for one, am quite ready and even anxious to agree to this, though I do not think it need be forgotten that some of these students will go to college. Also whenever I am called upon to make this agreement, and it is quite often these days, I want to follow it closely by calling attention to the fact that it is likewise not the mission of the public high school to prepare its students for the trades. It is the mission of the public high school to educate its students in the best sense of the word. I understand that to educate a student means to give him a knowledge of the culture of the world past and present, to school him into habits of accurate work, precise and clear thinking, in other words, to furnish him with the intellectual means for living his life as fully and appreciatively as possible. Incidentally this means that on leaving school he should have the power of further self-education. I do not think that during these years any direct account need be taken of the student's future need to earn money. This does not mean that I have any objection to vocational schools as such. On the contrary, I think that, in many cases, they are meeting a distinct need. If a boy of fourteen must learn a trade, or learn nothing, then it is much better for him to learn a trade. However it seems to me that we must think of this condition of affairs as unsatisfactory in a country like ours. We do not want to feel that we have any community or any class of people, whose children must be denied the privilege of a good high school education. In other words our educational ideals should be such, that if realized, the vocational school would no longer exist, or else its curriculum, like that of the college, would be based on a good secondary school education.

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* An address delivered before the Association of Mathematical Teachers in New England at Manchester, New Hampshire, February 27, 1915.

On the other hand I would not exclude any subject from the curriculum because it is practical. If it should be immediately useful to the student, so much the better, if it is only properly educative. In attempting to secure any particular result by a given mass of material, I think it is always in order to inquire if the same result could be obtained by the use of material which at the same time would be of more practical value. If domestic science as a subject has all the educational values of Latin, then domestic science should be substituted for Latin in all courses for girls.

What then should be the content of the curriculum of the secondary school? In considering this matter seriously, I find myself in agreement with such men as Sachs, who in his book "The American Secondary School" gives English, a foreign language, history, mathematics, and science as five subjects which he calls the constants of the secondary school course. The course may contain besides, courses in domestic science, manual training, stenography, etc. This is, I think, a proper view of the needs of the curriculum. There are many who seriously question the foreign language, and many who maintain that mathematics should not be required, or at least that the requirements should be very much modified. There are others who believe in mathematics, but wish to change the content of the course, and the methods of instruction, to such an extent that you and I would scarcely recognize the proposed courses as mathematical. The criticisms of mathematics as part of the curriculum, come from so many sources and are so insistent that it behooves us to take account of them, and it may well be that in so taking account we will be lead to better things for mathematics. It is good for any one to take account of himself occasionally and to justify himself and his activities. Such an extensive and careful consideration of these matters as has been undertaken recently by a committee of this association can not fail to be of permanent value.

The report of this committee when it appears will doubtless contain a very complete bibliography. I want to refer here to an article in the MATHEMATICS TEACHER of June, 1913, entitled "What Mathematical Subjects should be introduced into the curriculum of the Secondary Schools?" by William Betz of Rochester, New York. Mr. Betz gives many references to literature that are valuable to one desiring to know current tendencies.

Now in considering the matter of mathematics as a part of the secondary school course, what principles should guide us? In the first place it seems to me that we should be very conservative. The educational values of a given subject, and of given material in that subject are so hard to determine, that that which in the light of experience has proved to be reasonably good, should not be lightly thrown away. It should be discarded only for new material whose value has been definitely determined and found to be superior. On the other hand ultra-conservatism is likewise undesirable. We should remember that there have been changes in the method and the material of mathematical instruction in this country and elsewhere, and that these changes have not been accompanied by serious calamity. We do not spend much time extracting cube roots, neither do we insist that geometry be presented according to the letter of Euclid. We all agree, I think, that the change is good.

This is surely no time to fail to take account of the mass of criticism against mathematics and its teaching. There is now available for the curriculum such a large amount of material, in such a great variety of subjects, that that which remains must amply justify itself. Mathematics may be good to train the student in persistency and accuracy, but if that is all it does it may well have to resign its place in favor of another that can do this and more. That material will remain which can supply the most appropriate and unique combination of values. Doubtless we agree that this is the material which ought to remain.

We must seriously ask ourselves then whether the material we present and our methods of presentation are suited to the best evaluation of the subject. There are doubtless many different matters of expedience that must enter into this consideration. Students have many different needs, but we want to meet these needs by as few different courses as possible. And these courses must fit into each other in such a way that a student can change his mind as to his ultimate destination with a minimum loss.

Let us first ask ourselves then what are the values that we expect from the courses in mathematics. Not long ago the answer would have been: training of the logical powers. In the light of modern psychology we cannot lay too much emphasis on this, though lately even the psychologists seem to be admitting that the subject has a certain value of this sort. However at present we attach other values to mathematics. As I see it, mathematics has in the first place an historical value. It gives the methods and facts of a subject which has been at the foundation of progress in the physical sciences, and which to-day is more useful than ever. It certainly does give a training in careful accurate persistent work, the effect being upon the general character of the student, perhaps, rather than upon his logical faculties. The subject has a certain æsthetic value. It possesses certain elements of beauty that are unique. It gives to the student an opportunity to use his creative faculties in a way that scarcely any other subject does. Mathematics also has a tremendously practical value in that it is indispensable to the comprehension of much of the scientific learning of the day, and in that it is at the basis of all forms of engineering science. Thus we think of at least five values that attach to the subject, (*a*) the historical value, (*b*) the character-building value, (*c*) the æsthetic value, (*d*) an aid to understanding science, (*e*) the utilitarian value. Now how far do our present curriculum and methods of teaching, realize these values? I think we must see that so far as the teaching is concerned it does not do what it should. This is not possible with the present training that our teachers have. How can a teacher realize the historical value unless he himself is thoroughly acquainted with the history of mathematics and the allied sciences? How can he realize the æsthetic value unless he has pursued the study of mathematics long and successfully, and is indeed constantly interested in furthering his knowledge of the subject? He must himself be finding things that are new to him and be studying the approaches to problems for him unsolved, if he is to appreciate the interest and value of this sort of thing to the student. How can the teacher point with convincing confidence to the vast fields of application of the subject unless he has intimate acquaintance with the various fields of higher mathematics and a fair knowledge of at least physics and astronomy? It may well be that, to realize the logical training from geometry, all the equipment that is necessary on the part of the teacher is a keen mind and a familiarity with the subject itself. Such a teacher may even do much more, but if his knowledge stops here his teaching must fall far short of completeness. You will understand that I am criticising no one in particular. Teachers of mathematics, so far as I know them, are very conscientious and progressive, and are doing a very great deal with the material at their disposal. I am merely emphasizing what I consider to be a fundamental weakness in our present system. However it will be noted here that I am indicating a training as complete as that of most college and university teachers, and I think we shall have made great progress when we see that such a training is none too good for the teacher in the secondary school. We should look forward to the time when every teacher of mathematics has a good graduate training in mathematics. The college and university teacher would add to this a training in mathematical research, and the secondary teacher a training in the history and pedagogy of mathematics. However in the immediate sequel much could be done if only the mathematics in the secondary schools were taught by those who in college had at least a reasonable interest in the subject and who studied it there to some extent. Too often, especially in the smaller schools, the mathematics is taught by someone with no training or no fundamental liking for the subject, someone perhaps who is trained in English and history but who is given the mathematics because that is a subject anyone can teach. That teacher deserves only sympathy. I feel sure that much of the present criticism of mathematics in the schools comes, not from the subject-matter of the courses, but from the methods of teaching. Here then I think should be our first answer to criticisms at present directed against the curriculum, especially those criticisms that come from educators and school officials. We should say to them: "Do what you can to see that the conditions for the teaching of mathematics are as favorable as possible, and see to it that at least only those teach mathematics who want to do so. We will see to it that those who want to teach mathematics do so with a steadily improving equipment and with a greater efficiency. We can do this much better, if you will see to it that we do not teach more hours per week than is consistent with the best results. We are not sure that the subject matter of our courses is chosen to the best advantage but we do not think it should be changed except after careful study and experiment. This study and experiment we ourselves have in hand, and we shall be among the first to recommend change, when it is clear what change is desirable." We might add to this that in any case we can easily see to it that the student who passes our courses works with some steadiness and persistence, and that this might not be true in courses which some propose instead of mathematics. But this might be both irrelevant and impertinent.

As to the improvement of the equipment of the present teacher, that is something that every one of us can help to accomplish by improving himself. Some can make use of leave of absence, or of summer vacations to attend institutions of learning. Those who cannot do this can at least make use of the increasing volume of good literature available, to increase both technical and professional equipment.

But how shall we proceed in the matter of the improvement of the curriculum? We must depend upon the schools of education in the larger universities where educational experts, with the cooperation of skilled teachers, can experiment under proper conditions with new methods and new material which seem to be more desirable than those in current use. Perhaps, also, such experimentation may be carried out in the larger schools where there are many teachers and where proper classes for experiment may be formed.

Under present conditions then, with our ideals of secondary education, where is the curriculum at fault? What changes should we be thinking about? How shall we go about finding suggestions as to possible desirable changes? In the first place we can take account of the experience of other people in other countries. While it is proper and expedient to make use of the experience of other countries, we should in no case try to graft their system on our schools. Our conditions are peculiar to us, and our curriculum must be the one best calculated to meet our conditions. Many suggestions as to the improvement of the curriculum come to us from our combined experience in the teaching of the present curriculum. It is my final purpose to-day to mention some changes in the curriculum suggested to me by my thought as to the various purposes of the secondary school, and by my experience with students, and corroborated to some extent by the procedure in other countries.

In the first place, how about the relative proportion of the required and elective parts of the curriculum. I think the present proportion is certainly not far from right. It has been suggested that some sort of synoptic short course be given and that other mathematical courses be elective. I have no faith in this. Such a course would seem to me to be almost certainly valueless from every point of view. I do not see how it could be much more than a vague generality, which would even fail in requiring from the student a reasonable amount of wholesome effort. It is very hard to appreciate the facts and methods of mathematics without a certain mastery of technique. Teachers in college know how hard it is to give such courses even to students of considerable mathematical ability, and mastery of technique. No, I think that if the values of the subject are to be realized the required part of the course should consist of something more than half the whole. Then the question arises, is the material of the required course the best material. Here I am in serious doubt. With regard to algebra for example, I believe that the subject might be made somewhat less technical. I think the more difficult parts of it might be omitted, and emphasis placed more on the easy parts and their significance. Let us take for example the matter of factoring. Now it is quite desirable that the student should understand the general matter of factoring, and that he should be able to factor simple expressions. However I can not see that it is fundamental that he do more than this. If he meets a quadratic that he can not factor for example, it is not serious. Anyhow he will soon have at his disposal more powerful and direct methods which will produce the factors. Some little saving of time might be effected in this way, and at the same time the student would be relieved from an irksome task without apparent purpose. Then in geometry the work might be less formal. The introduction should make use of all the geometrical, mechanical, physical intuition that is available in the mind of the student. The introduction to the subject should be of an informal character. By means of drawing the student should be brought to comprehend the simpler figures and constructions in geometry, and by degrees be made to see the necessity of argument to establish geometrical truth. Steps have been taken in this direction, and I know of a text-book soon to appear that in a very definite and admirable manner, makes the approach to the subject in this way. We may say that this is not geometry, but I think it is. After all a proof is a relative matter. The formal proofs of Euclid are not so perfect that we need feel that we must never depart from them. If by a more informal procedure we can allay the student's suspicion that geometry is really a very dry and uninteresting subject, suitable only as a task-master for young boys and girls, and bring him to see that it is really a very simple, common, every-day subject, concerning things that are very real to him, I think there will be no reason to fear that his later appreciation of the rigors of the subject will be below par. We are not living up to our privileges if we fail to make use of anything that the student already has in his equipment. I do not think, however, that we should go to the extent of making geometry all or even nearly all of this character. There should be a formal phase of the subject based on the conceptions that have been clarified by the preliminary work and on a set of unproved propositions which includes all those which to the student's mind need no proof. I know of nothing more confusing to youth or adult than to try to prove that for which there is no evident need of proof. I know that some of us have fixed in our minds a certain kind of text-book that defines in a supposedly definite fashion the entities of geometry and gives a certain very small number of axioms the truth of which every one admits and then starts off by proving some propositions equally evident. I believe, however, that just as this text is an improvement over Euclid, other texts and methods may furnish further improvement. It seems to me also that the formal part of the geometry should be as simple and easy as possible. Only the simplest propositions should be given, and the originals should be very numerous but very easy. The methods and virtues of precise formal reasoning may be exhibited as well by easy propositions as by difficult ones. The student may be better trained by doing a large number of easy originals than by failing to do a few hard ones. There should, in any case, be many examples of the disastrous results of false procedure. For example in both algebra and geometry, the student should see by many examples that it is not safe to assume the truth of a given proposition, because it happens that using it as an hypothesis and reasoning from it in a perfectly correct fashion, he can derive a known conclusion.

In asking that fewer hard problems be given in algebra and geometry, I would have no notion of making the courses easier. What is lost in difficulty should be made up in thoroughness. If it worked out this way, the result would surely be a gain.

There are two serious changes which it seems to me might well be made in the required part of the curriculum. In the first place I think that the concept that we usually designate by the word "function" should be brought into the elementary algebra at the very beginning, and that every legitimate use of it should be made from then on. The graphical methods already in use in algebra would help to make this possible. This concept is one so useful everywhere in almost every aspect of human activity that I believe it would add a great deal to the value of the subject to the student. It is the very kind of thing that he will want to think about a great deal all his life. If he thinks about it clearly and accurately during his study of mathematics, it can not fail to be of use to him. That is, here is a place where the faculties of the student may be trained in the very form of activity that is to engage them in the sequel, independent of what he may do for a livelihood. Then also the function concept furnishes a unifying principle on which to build much of the elementary and advanced mathematics and helps to lend both concreteness and coherence to the subject. I do not mean that a fetish is to be made of the function concept, and that there should be an effort made to connect everything with it, whether it will or no. Such a fad tends to bring a notion into disrepute and properly so. Some years ago when graphical methods were being introduced into the secondary courses and into the text books, there was a desire on the part of some to make graphical methods account for everything. Now we are using graphical methods increasingly, and no doubt we all agree that they are good. But other methods are good and we shall retain them. We will be graphical when to be graphical suits us best, but we shall feel free to use other methods when they seem to be best adapted to the matter in hand.

Then I think it would be very desirable to add to the required part of the curriculum the elements of trigonometry, not as a distinct subject, but as a part of algebra and geometry. I do not mean the whole subject of trigonometry as found in our text-books, but its simplest portions. For example in algebra, with the rectangular coordinate system at our disposal it is very easy and natural to define the trigonometric functions of the general angle and derive the simple relations amongst them. Simple trigonometric identities can be derived. In geometry use can be made of the functions of an angle, the law of sines and the law of cosines may be developed, and a part of the course in geometry may consist of the solution of triangles, right and oblique, by use of the natural functions. The student can be supplied with simple instruments and led to measure and compute heights and distances. I understand that all this needs to be very simple, but it can be simple and at the same time very interesting and instructive. From the standpoint of the various values to be expected from the required course in mathematics, it seems to me that it is a distinct mistake not to make use of this material, so easily available. The historical setting of the subject is very interesting, and very easily appreciated. As an aid to progress in other science and from a purely utilitarian viewpoint, the subject is very valuable. The concreteness of much of the subject matter and its power in solving certain types of interesting problems, can not fail to make a strong appeal to every student. It seems to me that from every point of view it would be desirable if possible to include this material in the required course. You will understand that I am not here especially considering my convenience as a teacher of college mathematics. If I did that, I might recommend that every moment possible be spent in drilling the student in the use of exponents, positive, negative, integral, and fractional; that the student be properly coached in the simplification of all kinds of complex and inhuman fractions, and that he be able to solve automatically any equation that may be reduced to the quadratic form. However, even if I were thinking of the matter most selfishly, I do not believe I would recommend such a procedure. The boy or girl is likely to be most teachable who has from the beginning the best appreciation of the significance of the subject matter which he studies and his training in technique may fairly be distributed among the various stages of his student career.

Then how about the elective part of the course? Is the material for that well selected? It seems to me that here is our greatest weakness. The elective work is made up of three relatively distinct subjects each of them pursued for a half year, and one of them is itself rather an incoherent conglomeration. The material of this portion of the course should be chosen so as to best realize all the values of the subject, but with special attention to the role of mathematics as an aid to science and to its utilitarian value. It happens that, in my opinion, the material which would best realize these values would also best realize all the values which we have attached to the subject. With the graphical methods at our disposal, and with the function concept well developed in the required work, what could be more natural than to develop around these a unified course that might well include logarithms, some further study of trigonometry, selected portions of our present college algebra, and the very elements of analytic geometry and calculus. By the terms analytic geometry and calculus I do not signify the traditional college courses in these subjects. By analytic geometry I mean merely the simple and fundamental portions of that subject, such as would follow very readily and easily from the knowledge of the rectangular coordinate system which the student has already in his possession. By calculus I mean merely the notion of the slope of the tangent to a curve, the rate of change of one variable relative to another, the area under a curve, the evaluation of the limit of the sum by means of a definite integral. These ideas can all be illustrated by functions of a simple character. The functions chosen for illustration and for problems could be as simple as experience might indicate to be proper. Much of the technique of the subject, as now usually taught, could be avoided and yet a relatively clear notion of the character of the subject and the field of its application could be obtained. Plenty of simple problems are available for practise and they would be of great value if each one of them were carefully and thoroughly studied with due emphasis on the geometrical and mechanical significance of each.

As I have already indicated, it seems to me that a course of this kind would be a considerable advantage over the present elective course. It is a more unified course and success in it would mean more continuous application and appreciation. It gets the student to a place where he can see to a much better advantage the dignity and far reaching power of mathematics. He can much better appreciate the genuine usefulness of the subject. If further schooling be denied him he is better able to study for himself. If his interests and work demand it, he can acquire by himself a considerable knowledge of mathematics or of physics, or of the elements of engineering. This is especially true when it is considered that students taking elective mathematics would likely take physics, and such other courses in science as were available. From the standpoint of the historical value of the subject of mathematics it is a shame to omit from a course which will be many a capable and interested boy's last chance at the subject, a topic which has had so important an influence on almost every aspect of world activity as the differential and integral calculus, even though it be admitted that only the most elementary and simple aspects of the subject can be included.

In view of the agitation in favor of omitting the incommensurable case in geometry one might hesitate to introduce the notion of the slope of a curve. Both make use of the notion of limit. However I think the slope of a curve is much the easier notion. It can be approached in a definite and concrete manner and the student will readily appreciate the character of the problem involved. I think that in the incommensurable case many students fail to see the character of the problem, or that there is any problem.

It will be noted that I have said nothing about solid geometry. I would not like to see solid geometry omitted. I would almost be willing to see it omitted if my other desires could be realized. However I see no reason why, since it is an elective anyhow, it could not be given as an extra subject for, say, two periods a week for a year. It seems to me to be not unwise even in the secondary school to permit students to specialize to a limited extent in the direction of their major interests. I would distribute the subject through a year because I feel its principal unique value, aside from a few formulas in mensuration, lies in the cultivation of the space intuitions. That would be best accomplished by a slower development through a longer period.

It will also be offered in criticism of my suggestions, that the material I have indicated will certainly require a longer time than now available for mathematics. This may be true. I am not sure of it. I know of no way to be sure except to work out the material in detail and give it several actual trials. I think there might be some point to such a criticism. However there are very many forces at work to extend the period of secondary instruction and it is rather clear to many people that in one way or another this period will be lengthened to five or six years. I believe we should lend our support to this as being desirable from every point of view. In my opinion this change is as likely and as imminent as is any serious change in our mathematical curriculum. Therefore the two things may be considered together. The longer period of secondary instruction would simplify many difficulties of the curriculum.

In conclusion, then, may I offer a brief résumé of what I have said.

1. Any serious change in the curriculum should be made only after most careful consideration based on skilled experimentation. In a matter of this kind leaders in education ought to be truly conservative.

2. Much of the criticism against the curriculum will disappear in the face of an increasingly careful and thorough preparation of teachers.

3. In spite of our conservatism we should be awake to every possibility of improvement, realizing that only thus will there ever be even a suggestion for improvement. Our conservatism should operate only to prevent the final adoption of any reform measure without due consideration and experimental evaluation.

4. A short synoptic course at beginning of high school is not feasible.

5. The curriculum at present is very good. Of possible changes that we may look forward to, I think that those most deserving of consideration are:

(*a*) The omission of some of the more difficult technique from algebra.

(*b*) The informal, intuitional approach to plane geometry with perhaps some abbreviation of that subject by omitting unnecessary propositions and difficult originals.

(*c*) The introduction of the function concept near the beginning of the required course and its use in the sequel both for its own sake and as a basis of unification.

(*d*) The introduction of the elements of trigonometry into the required course.

(*e*) The unification of the main part of the elective course around the function concept and the notion of the coordinate system including the elements of analytic geometry and calculus.

Some of these changes may of course be incorporated into the present curriculum without serious alterations. Some of them are already being incorporated. Others would require great change in point of view and method and might require a longer period of instruction than is now allotted to mathematics. While after considerable reflection I believe all the changes suggested would be desirable, I also believe that no one of them or any other should be adopted until by actual experiment, by skilled and open-minded teachers, it has been proved to be really desirable.

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